Random Stuff 2

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$\mathbb{Z} \to$ integers $\mathbb{Q} \to$ rationals, $\mathbb{R} \to$ reals, $\mathbb{C} \to$ complex

$\mathbb{Q}^c \to$ irrational

$\mathbb{Z} \subset \mathbb{Q}$ $\mathbb{R} \subset \mathbb{C}$

$\Rightarrow$ Take two rationals q_1, q_2 where $q_1 < q_2$.

q_3 (rational) q_3' (irrational) $\to$ There will infinite irrational and rationals between q_1 and q_2

$(a, b) \subset \mathbb{R}$

smallest and lar – element in (a, b) is not defined

$\sup\{(a, b)\} = b$ $\inf\{(a, b)\} = a$

$\sup\{[a, b)\} = b$ $\inf\{[a, b)\} = a$

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$\sup\{(a, b), (b+1, c)\} = c$

$|(0,1)| = \underbrace{|\mathbb{R}|}_{\text{uncountable}}$ $\underbrace{|\mathbb{N}|}_{\text{countable}} < |\mathbb{R}|$

$n_2 \in \mathbb{N}$
$n_1 \in \mathbb{N}$
$(n_2 - n_1) \in \mathbb{N}$

$f : (0,1) \to \mathbb{R}$ can be a bijection

$f(x_1) = y_1$ if $y_1 = y_2 \Leftrightarrow x_1 = x_2 \to$ one-one
$f(x_2) = y_2$

$f(x_1) = y_1$ if $y_1 = y_2 \iff x_1 = x_2 \rightarrow$ one-one
$f(x_2) = y_2$

and $f^{-1}(y_1) = x_1$ exists $\forall y_1 \in \mathbb{R}$
onto

Suppose $f : \mathbb{N} \rightarrow \mathbb{R}$ is a bijection (one-one and onto)

Intuition

$f(x_1) = f(x_2) \iff x_1 = x_2$

$\forall y \in \mathbb{R}$, $\exists\, x \in \mathbb{N}$ such that $f(x) = y$

$\Rightarrow \mathbb{R} = \{ f(1), f(2), \ldots \}$

$\rightarrow$ countable but $\mathbb{R}$ is uncountable $\Rightarrow\Leftarrow$

$|P(A)| > |A|$

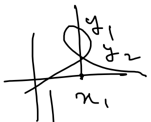
$\rightarrow$ a set that contains all subsets of itself

$A = \{1, 2, 3\}$ $P(A) = \{ \{\phi\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\},$
$\{3, 1\}, \{1, 2, 3\} \}$

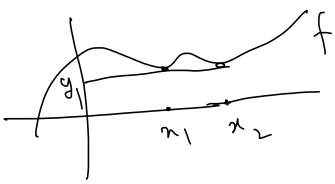
To see a formal proof we need Cantor's Diagonal Argument

$\mathbb{R}^2$

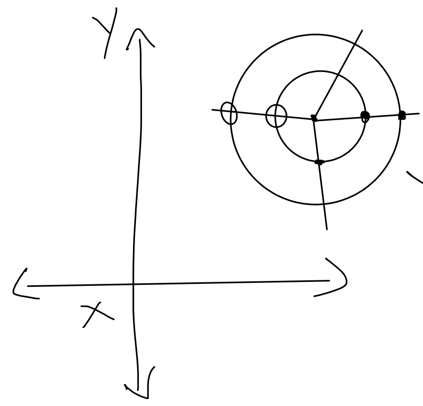
$[0, 1] \not\cong \mathbb{R}$



f is not a function



f is a function



$\rightarrow$ Both circles are of same cardinality

$f : X \rightarrow Y$
domain, codomain

$f(X)$ is the range

$f(X)$ may not be equal to Y

In function $f(x) = y_1$ and $f(x) = y_2 \Rightarrow y_1 = y_2$

In function $f(x)=y_1$ and $f(x)=y_2 \Rightarrow y_1=y_2$
(basic difference with map)

$f(x_1)=y_1$, $f(x_2)=y_1 \not\Rightarrow x_1=x_2$ in function